

MST125

TMA 02

2018J

Covers Units 6, 7 and 8

Cut-off date 6 March 2019

You can submit this TMA either by post to your tutor or electronically as a PDF file by using the University's online TMA/EMA service.

Before starting work on it, please read the document *Student guidance for preparing and submitting TMAs*, available from the 'Assessment' area of the MST125 website.

The work that you submit should include your working as well as your final answers. Your solutions should not involve the use of Maxima, except in those parts of questions where this is explicitly required or suggested. Your solutions should not include the use of any other mathematical software. If you have a disability that makes it difficult for you to attempt any of these questions, then please contact your Student Support Team or your tutor for advice.

Your work should be written in good mathematical style, as demonstrated by the example and activity solutions in the study units. You should explain your solutions carefully, using appropriate notation and terminology, and write in sentences. As usual, you should simplify algebraic answers where possible. Five marks (referred to as good mathematical communication, or GMC, marks) on this TMA are allocated for how well you do this.

Your score out of 5 for GMC will be recorded against Question 10. You do not have to submit any work for Question 10.

Question 1 – 15 marks

You should be able to answer this question after studying Unit 6.

- (a) For each of the following linear transformations, write down its matrix and describe the transformation.

(i) $g(x, y) = (5x, 2y)$

(ii) $h(x, y) = (x + 3y, y)$

(iii) $k(x, y) = (y, x)$ [5]

- (b) Use the matrices that you found in part (a) to show that the linear transformation $f = k \circ h \circ g$ is represented by the matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 2 \\ 5 & 6 \end{pmatrix}. \quad [2]$$

- (c) Show that f is invertible, and find the matrix that represents f^{-1} . [2]

- (d) Find the equation of the image $f(\mathcal{C})$ of the unit circle $\mathcal{C}$, in the form

$$ax^2 + bxy + cy^2 = d,$$

where a , b , c and d are integers whose values you should find. [5]

- (e) Calculate the area enclosed by $f(\mathcal{C})$. [1]

Question 2 – 10 marks

You should be able to answer this question after studying Unit 6.

- (a) The isometry f maps the points $(0, 0)$, $(1, 0)$ and $(0, 1)$ to the points $(4, 3)$, $(4, 4)$ and $(5, 3)$, respectively.

- (i) Determine f in the form $f(\mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{a}$, where $\mathbf{A}$ is a 2×2 matrix and $\mathbf{a}$ is a vector with two components. [2]

- (ii) Find the fixed points (if any) of f , and hence state whether f is a translation, rotation, reflection or glide-reflection. [3]

- (b) The transformation k is the reflection in the line $y = -x + 4$.

By using the translation h that maps the point $(0, 4)$ to the origin, and its inverse h^{-1} , determine k in the same form as you found f in part (a)(i). [5]

Question 3 – 15 marks

You should be able to answer this question after studying Unit 7.

- (a) Find the partial fraction expansion of the rational expression

$$\frac{3x^3 + 11x^2 - 9x - 15}{x^2 + 3x - 4}. \quad [10]$$

- (b) Use the `partfrac` command in Maxima to verify your answer to part (a). Include a screenshot or printout of your Maxima worksheet in your answer. [1]

- (c) Hence (without using Maxima) find the integral

$$\int \frac{3x^3 + 11x^2 - 9x - 15}{x^2 + 3x - 4} dx. \quad [4]$$

Question 4 – 15 marks

You should be able to answer this question after studying Unit 7.

Consider the function $f(x) = \frac{x^2 + 5x + 7}{x + 2}$.

- (a) Find the domain and intercepts of f . [2]
- (b) Find $f'(x)$. [2]
- (c) Find the coordinates of any stationary points of f . Construct a table of signs for $f'(x)$, determine the intervals on which f is increasing or decreasing, and determine the nature(s) of the stationary point(s). [5]
- (d) Find the equations of the asymptotes of f . [2]
- (e) Determine whether f is an even or odd function, or neither. [1]
- (f) Sketch the graph of f . [3]

Question 5 – 15 marks

You should be able to answer this question after studying Unit 7.

- (a) Use the identity $\cosh^2 x - \sinh^2 x = 1$ to find the integral

$$\int \sinh^5 x \, dx. \quad [7]$$

- (b) Use your answer to part (a) and a hyperbolic substitution to find the integral

$$\int \frac{x^5}{\sqrt{x^2 + 9}} \, dx. \quad [8]$$

Question 6 – 5 marks

You should be able to answer this question after studying Unit 8.

Consider the differential equation

$$\frac{dx}{dt} = \frac{t}{\sqrt{t^2 + 4}}.$$

- (a) Which of the methods described in Unit 8 for finding solutions of differential equations would you use to solve this equation? Give a reason for your choice. [1]
- (b) Find the general solution of the differential equation, expressing x explicitly as a function of t . [3]
- (c) Hence find the particular solution of the differential equation that satisfies the initial condition $x(0) = 3$. [1]

Question 7 – 5 marks

You should be able to answer this question after studying Unit 8.

Consider the differential equation

$$\frac{dy}{dt} = \frac{\sinh t}{e^y}.$$

- (a) Which of the methods described in Unit 8 for finding solutions of differential equations would you use to solve this equation? Give a reason for your choice. [1]
- (b) Find the general solution of the differential equation, expressing y explicitly as a function of t . [3]
- (c) Hence find the particular solution of the differential equation that satisfies the initial condition $y(0) = 2$. [1]

Question 8 – 5 marks

You should be able to answer this question after studying Unit 8.

Consider the differential equation

$$\frac{dy}{dx} - \frac{5y}{x} = 1 \quad (x > 0).$$

- (a) Which of the methods described in Unit 8 for finding solutions of differential equations would you use to solve this equation? Give a reason for your choice. [1]
- (b) Find the general solution of the differential equation, expressing y explicitly as a function of x . [4]

Question 9 – 10 marks

You should be able to answer this question after studying Unit 8.

The intensity of light I , measured in luxes (lx), at depth y (in m) below the surface of a lake can be modelled by the differential equation

$$\frac{dI}{dy} = -kI \quad (I > 0, y \geq 0),$$

where k is a positive constant.

- (a) Find the general solution of this differential equation in explicit form. [2]
- (b) The intensity of light at the surface of the lake is 2000 lx. Find the particular solution that describes the intensity of light as a function of depth below the surface of the lake. [2]
- (c) The intensity of light is 500 lx at 10 m below the surface of the lake. Use this fact to find the value of the constant k . Give both its exact value and its value to two significant figures. [2]
- (d) Use your particular solution and the exact value of k to find the intensity of light 17 m below the surface of the lake. Give your answer in luxes to two significant figures. [2]
- (e) Use Maxima to find the solution of the initial value problem

$$\frac{dI}{dy} = -kI, \quad \text{where } I(0) = 2000.$$

Include a screenshot or printout of your Maxima worksheet in your answer. [2]

Question 10 – 5 marks

Five marks on this assignment are allocated for good mathematical communication in Questions 1 to 9.

You do not have to submit any extra work for Question 10, but you are advised to check through your assignment carefully, making sure that you have explained your working clearly, used notation correctly, written in sentences and rounded answers as requested. [5]
